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AN APPLICATION OF ROBUST FILTERS IN ECG SIGNAL PROCESSING

Robust filtering is very promising area in application of biomedical signal processing. Signals are usually recorded with noise which has various character from baseline wander to very impulsive nature. The robust technique has been recently proposed as the tool to eliminate outliers in data samples. The main purpose of this paper is to present the mean-median filters in an application of ECG signal processing. The presented filter is evaluated in the presence of a real EMG noise and a simulated impulsive noise as a Gaussian-Laplace sequence. In order to suppress a noise with the best possible means, the special expression is proposed. The measure of distortions which are introduced to a signal after operation of filtering is estimated by using the normalized mean square error. This factor is used to compare an operation of considered filters. Experimental results show improved performance according to the reference filters.

1. INTRODUCTION

Linear filtering technique is commonly used in many areas of digital processing. The main assumption of this technique is that a noise is characterized by Gaussian distribution. Such approach is justified by the Central Limit Theory and in addition, the analytical form of solution is often obtained [11]. Non-gaussianity often results in significant quality degradation for systems optimised under the Gaussian assumption [11]. Such systems are very sensitive to the presence of outliers. For example the mean filter is optimal filter for Gaussian noise in the sense of mean square error, but performs poorly in the noise which is described by heavy-tail distributions. This is the reason to investigate non-linear filtering alternatives [3]. The non-linear filters are characterized by their robustness to impulsive noise. The most interesting are filters which belongs to the class of M-filters. Such filters are a sliding window filters and the output of a window is estimated as the maximum likelihood estimation of location [2,6].

Biomedical signal processing requires the use of filters to shape the frequency content of the signal. Signal smoothing, enhancing or shape preserving in the situation, that an impulsive noise appears, it makes that only alternative for the linear filtering is using a robust methods. Linear filters tend to blur sharp edges, destroy lines and other fine image or signal details in the presence of heavy tailed noise. Whereas there is an important class of smoothing applications that requires careful treatment and preservation of signal edges [3]. This required robust filtering methods. The

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biomedical signals are recorded in noisy environment. The sources of disturbances are different kind of operating devices in the human environment and man is also a source of noise. In the biomedical systems the first step of processing of biomedical signals is very important, because all later activities depend on the quality of the first step which usually apply the noise reduction algorithms [8]. Because there exists many different biomedical signals, the electrocardiogram (ECG) signal is chosen, which can be disturbed by: 50 Hz power line interference, baseline wander, motion artefact, electromyogram (EMG). In fact, most types of noise are not stationary, it means, that the noise power measured by the noise variance features some variability. The EMG contaminations in ECG signals distort low-amplitude ECG wave components and hence lower the accuracy of computer-aided measurements of various morphological characteristics [4]. The muscle noise is the most difficult noise that should be suppressed, because the spectra of EMG signal overlap for a wide range of frequency the spectrum of ECG signal [12]. A white Gaussian noise is usually used to model an EMG signal, but the muscle noise shows frequently an impulsive nature, and it means that the Gaussian model may fail. Another model which can described some cases of a muscle noise is an application of symmetric α-stable distribution [9].

The main aim of this paper is to present the mean-median, robust filter (MEM filter) which can effectively suppress a muscle noise and an impulsive type of noise. The second aim is to check the possibility of estimating the “tuning” parameter λ of MEM filter with respect to a noise level. The paper is organized in the following way. In the next section the theory of the robust filtering is introduced and the mean-median filter is presented. In the Section 3, the method of evaluation is presented, some results and discussion. Some conclusions are presented in the last section. The reference filters are the moving averaging filter, the myriad filter and the median one.

2. THE ROBUST FILTER

Consider the desired signal $s(n)$ disturbed with noise components $v(n)$ and then the input signal $x(n)$ has a form $x(n) = s(n) + v(n)$. The main aim of filtering is to estimate the signal samples $s(n)$ by using the noisy samples $x(n)$. The class of M-filters is the running window filter outputting the M-estimator (maximum likelihood estimator) of location of the elements in the moving window. Assume that the measurement errors are distributed according to nongaussian distribution. The maximum-likelihood formula for the estimated parameters $\hat{\beta}$ which are predicted values of $s(n)$, can be written as:

$$P = \prod_{i=1}^{N} \exp\left[-\rho(x_i - \beta)\right] \tag{1}$$

where the $\rho(\cdot)$ function is so called the cost function [2]. The properties of M-estimators depend on properties of the cost function. Taking the logarithm of (1), obtained expression should be minimized:

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{N} \rho(x_i - \beta) \tag{2}$$

where $\arg\min_{\beta}(\cdot)$ denotes the value of β that minimizes the expression in parenthesis [2,6] and the $\rho(z)$ is a function of a single variable $z \equiv (x_i - \beta)$. Let the function $\psi(z)$ be the derivative of $\rho(z)$, i.e., $\psi(z) = d\rho(z)/dz$. The $\psi(z)$ (called the influence function) function is some odd, continuous, and sign-preserving function [7,10].

The special case of M-filters are the mean filter and the median one. When the errors in measurements are normally distributed, i.e., $Prob\{x_i - \beta\} \sim \exp[-(x_i - \beta)]$ then optimal estimator has the form $\rho(z) = 0.5 \cdot z^2$ and $\psi(z) = z$. These last dependence leads to the sample mean filter which is optimised under the normal distributed errors and reduced to the standard least-squares estimation. When the errors in measurements are distributed as a double or two-sided exponential, i.e., $Prob\{x_i - \beta\} \sim \exp[-|x_i - \beta|]$ then $\rho(z) = |z|$ and $\psi(z) = \mathrm{sgn}(z)$. This expression denotes the median filter. Some properties of the median filter are described in [7,13].

Robust estimation is the means to solve the problem when the distribution function is in fact not precisely known. In this case, an adequate approach is to assume, that the density function is a member of some set, or some family of parametric families, and to choose the best estimator for the least factorable member of that set [3]. The most commonly used form in modelling outliers for detection and robustness studies is the two-component mixture, where both distributions are zero mean, but one has a greater variance than the other [3]. Using this facts, assume that the noise probability distribution is scaled version of a known member of the P_ε family of ε - contaminated normal distributions proposed by Huber [5] $P_\varepsilon = \{(1-\varepsilon)\Phi + \varepsilon H : \quad H \in S\}$, where Φ is the standard normal distribution, S is the set of all probability distributions symmetric with respect to the origin (i.e., such that $H(-x) = 1 - H(x)$), and $\varepsilon \in [0,1]$ is the known fraction of "contamination". The presence of outliers in a nominally normal sample can be modelled by a distribution H with tails that are heavier than that of normal distribution. Now let Φ denotes Gaussian distribution $N(0, \sigma_G^2)$ with variance σ_G^2 and H is Laplacian (or double-exponential) $L(0, \sigma_L^2)$ with variance σ_L^2 [1,3], then Gaussian is in the center and Laplacian in the tails and switches from one to the other at a point whose value depends on the fraction of contamination ε, larger fractions corresponding to smaller switching points, and vice versa [1,3]. Another method which is frequently applied in digital signal processing to model the impulsive noise is the family of the symmetric α-stable distributions (SαS) [11]. This model is not used in this work.

As a consequence of above study, a convex combination of the mean and the median filters (MEM) can be defined as [3]:

$$y(n) = (1-\lambda)x_{\mathrm{ave}}(n) + \lambda\, x_{\mathrm{med}}(n), \qquad \lambda \in [0,1] \tag{3}$$

where $x_{\mathrm{ave}}(n)$ is the output of mean filter and $x_{\mathrm{med}}(n)$ is the output of median filter calculated in moving window of size $N = 2k + 1$ and are defined as:

$$x_{\mathrm{ave}}(n) = \arg\min_{\beta} \sum_{i=n-k}^{n+k} \left(x(n+i) - \beta\right)^2 \quad \text{and} \quad x_{\mathrm{med}}(n) = \arg\min_{\beta} \sum_{i=n-k}^{n+k} \left|x(n+i) - \beta\right| . \tag{4}$$

As a useful quality factor for a robust estimator, Huber suggests its asymptotic variance since the sample variance is strongly dependent on the tails of the distribution. The asymptotic variance is defined as:

$$V(z,F)=\int(\psi(z))^2\,dF(z) \tag{5}$$

where $\psi(z)$ is the influence function and $F(z)$ is the common distribution function of the input with corresponding $f(\theta)$ as the density function. Using the influence functions for the mean and the median filter, the influence function for the MEM filter is given as $\psi(z)=(1-\lambda)z+\lambda\,\mathrm{sgn}(z)$.

As was proof in [3] the asymptotic variance for MEM filter is defined as:

$$V(\mathrm{MEM},F)=(1-\lambda)^2\mu_2+\frac{\lambda^2}{4f(\theta)}+\lambda(1-\lambda)\frac{\mu_1}{f(\theta)} \tag{6}$$

where $\mu_k=E|X-\theta|^k$, $k=1,2$ are the central moments. Using (14) the expression for optimal value of $\lambda_{\min}$ is given as [3]:

$$\lambda_{\min}=\left(\mu_2-\frac{\mu_1}{2f(\theta)}\right)\Bigg/\left(\mu_2+\frac{1}{4f(\theta)^2}-\frac{\mu_1}{f(\theta)}\right) \tag{7}$$

When the input is Gaussian, the mean filter leads to better results of filtering than the median filter in suppression Gaussian noise and the $\lambda_{\min}=2/(2+\pi)$. Likewise if the noise is Laplacian, then median filtering tends to obtain better results of filtering than the mean filter, and then $\lambda_{\min}=2/3$. It is worth noting than parameter λ can change the MEM filter from linear (mean filter) to non-linear, robust filter (median filter).

3. EXPERIMENTAL RESULTS

The presented MEM filter is evaluated using the normalized mean square error defined as: $\mathrm{NMSE}=\sum_{i=1}^{N}[y(i)-s(i)]^2/\sum_{i=1}^{N}[s(i)]^2\cdot 100\%$ where: y*(i)* is the output of the myriad filter, *s(i)* is the deterministic part of signal, without a noise and *x(i)* is the noisy signal. The NMSE factor is the distortion measure of a signal after filtering. For the testing purpose the pure ECG cycles (i.e. with high value of SNR) are generated using linear combination of Hermite functions on the base of real ECG cycles sampled at 2kHz. For testing 5 different shapes of ECG cycles are chosen, each of length 1560 samples. Then the noise samples are added to ECG cycles with known value of the standard SNR factor (5, 10, 20 and 30 dB). In this work a simulated noise and a real electromyogram samples (sampled at 2kHz) are used. The mixture ε-contaminated (ε = 0.4 [1]) Gaussian $N(0,1)$ and Laplacian $L(0,\sigma_L)$ noise with value of $\sigma_L^2=1,2,\text{and}\,4$ are applied as artificial noise. The NMSE factor is calculated for 200 different realizations of noise and then average value of NMSE is calculated.

The values of NMSE factor are calculated for three values of λ. At first value of λ is optimal for Gaussian noise, at second value of λ is optimal for Laplacian noise. And at the third case for the optimal value of λ_{opt}, when the NMSE gets the minimum value. But in this case the knowledge of clean ECG cycle is required. This is not possible in real live measurements. In order to estimate λ'_{opt} only on the base of input signal and a noise level, two additional parameters are introduced. These are the kurtosis and the first ordinary moment m_2 calculated as $m_2 = \frac{1}{N}\sum_{i=1}^{N}[x(i)]^2$. Then λ'_{opt} can be calculated as the nonlinear expression which depends on the kurtosis and m_2 as:

$$\lambda'_{opt} = 0.3 - 0.03 \cdot m_2 - 0.04 \cdot \text{kurtosis} + 0.12 \cdot m_2^2 - 0.05 \cdot m_2 \cdot \text{kurtosis} + 0.007 \cdot (\text{kurtosis})^2 \qquad (8)$$

The results for mixture noise and the real muscle noise are presented in Table 1 and Table 2 respectively. The reference filters are the mean, myriad and median filters.

TABLE 1. Average NSME factor of 200 trials for a mixture ε-contaminated Gaussian and Laplacian noise (length of filter moving window N = 21).

SNR [dB]	myriad filter (k = 1)	moving average	median filter	MEM filter (λ_{opt})	MEM filter (λ'_{opt})	MEM filter λ=2/(2+π)	MEM filter λ=2/3
$\sigma_r^2 = 1$							
5	1.1635	1.3205	1.6444	1.1488	1.1817	1.1994	1.3422
10	0.4749	0.6734	0.6244	0.4534	0.467	0.4663	0.5119
20	0.1503	0.4211	0.0941	0.0831	0.0921	0.1031	0.0889
30	0.1328	0.3943	0.0306	0.0303	0.0492	0.0703	0.0431
$\sigma_r^2 = 2$							
5	1.2167	1.3728	1.3858	1.1294	1.1417	1.1349	1.1901
10	0.4554	0.713	0.4921	0.4033	0.4103	0.4078	0.4217
20	0.1404	0.4135	0.0781	0.0712	0.0826	0.0920	0.0761
30	0.1355	0.4379	0.0284	0.0282	0.0498	0.0705	0.0421
$\sigma_r^2 = 4$							
5	1.1694	1.3283	1.0861	0.9696	1.0132	0.9957	0.9797
10	0.4436	0.6748	0.4138	0.3593	0.3728	0.3729	0.3681
20	0.1473	0.3996	0.0764	0.0694	0.0793	0.0937	0.0755
30	0.1254	0.3759	0.0281	0.0278	0.0461	0.0659	0.0401

TABLE 2. Average NSME factor of 200 trials for a muscle noise (length of filter moving window N = 21).

SNR [dB]	myriad filter (k = 1)	moving average	median filter	MEM filter (λ_{opt})	MEM filter (λ'_{opt})	MEM filter λ=2/(2+π)	MEM filter λ=2/3
5	5.3004	4.9837	6.4943	5.2451	5.3536	5.4067	5.7611
10	1.7512	1.8411	2.1366	1.7181	1.7719	1.7579	1.874
20	0.2794	0.5105	0.2635	0.222	0.2286	0.2358	0.2336
30	0.1436	0.4102	0.0472	0.0456	0.0621	0.0818	0.0564

The best results of filtering (the smallest value of NMSE factor), i.e., the smallest distortion in the filtered signal are obtained in all cases of change SNR and variances of Laplace part of a noise for MEM filter when λ is chosen optimally. But disadvantage of such selection is the requirement of acquaintance of “pure” signal. In ambulatory measurements of ECG signal such condition is not possible. An operation of MEM filter with estimated value of λ'_{opt} leads to obtained a little worse results than the optimal MEM filter results and MEM filter with λ = 2/3 and λ = 2/(2+π).

In the case of muscle noise the obtained results are not such optimistic. When the SNR is low, i.e., SNR=5 dB, the best results are obtained for moving average filter. For $\mathrm{SNR} \geq 10\,\mathrm{dB}$, the MEM filter with optimal value of λ_{opt} introduces the smallest distortions in filtered signal. The results obtained for MEM filter with λ'_{opt} parameter estimated on the base of m_2 and kurtosis are near to optimal λ_{opt} except for SNR=30 dB.

4. CONCLUSIONS

The mean-median filter (MEM filter) with choice of λ in this paper is presented. The analyzed filter evaluation is motivated from robust statistics, particularly the possibility of model the muscle noise with a mixture ε-contaminated Gaussian and Laplacian noise. The usefulness of applying the MEM filter is statistically analyzed through the measurements of distortion after filtering with respect to a “clean” signal. The nonlinear combination of m_2 and kurtosis is proposed to obtain value of λ parameter which is practically optimal for filter action.

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