Computer-aided image processing,
analysis of textures,
morphological spectra

Juliusz Lech KULIKOWSKI, Małgorzata PRZYTULSKA, Diana WIERZBICKA*

APPLICATION OF MORPHOLOGICAL SPECTRA TO COMPUTER-AIDED ANALYSIS OF TEXTURES

Abstract: A method of computer-aided analysis of textures based on morphological spectra is presented. The components of morphological spectra are obtained as the result of four basic linear operations and permutations performed on a bitmap representing the image. The system of morphological spectra can be represented by a tree whose root corresponds to the analyzed image, while nodes of any given level represent spectral components of the corresponding level of morphological structure of the texture. Spectral components assigned to any fixed tree level contain complete information about the analyzed image. There are described formal properties of morphological spectra as tools for a multi-level characterization of textures. For characterization of textures under consideration according to their morphological properties only selected branches of the tree can be taken into account. The similarity of morphological spectra is thus the basis of discrimination of textures. Practical application of morphological spectra to texture recognition is illustrated by a numerical example.

1. INTRODUCTION

Analysis of textures plays a significant role in numerous computer-aided image processing areas. It is a basic step to a selection of regions of interest (ROI) in medical examination of images of human organs or of biological tissues. For this purpose numerous methods based on morphological, statistical, fractal and many other approaches have been proposed [1,4,5,6]. The main difficulty in texture analysis consists not only in a large variety of textures occurring in biological or medical examinations but also in the fact that certain types of textures are characterized by a multi-level morphological structure. E.g., on the lowest level of a biological specimen examination a grainy structure can be observed, on the middle level the grainy structures form clusters and on the highest level the clusters are ordered within elongated strips. Typical texture analysis methods: morphological, statistical, etc. are rather ineffective in description of such structures, spectral and fractal methods, due to their multi-scalar properties, suit better to it. That is why in [2] a method of texture analysis based on a set of multi-level image transformations has been proposed. The method then has been simplified due to introduction of a concept of multi-level morphological spectra obtained as the result of sequences of standard linear operations and permutations [3]. In this paper the (slightly modified) method is shortly presented and illustrated by numerical examples.

2. MORPHOLOGICAL SPECTRA

* Institute of Biocybernetics and Biomedical Engineering PAS,4 Ks. Trojdena Str, 02-109 Warsaw, Poland

An image is assumed to be given in the form of a rectangular bitmap. Textures visible in the image are analyzed and recognized within testing windows of a $K\times K$-size, where $K=2^k$, k being a fixed natural number. A selected part of the bitmap representing the part of image within a current testing window will be denoted by $S^{(0)}$. It has the following form:

Formally, $S^{(0)}$ is considered as a 0-th level (initial) morphological spectrum and it will be

$$\boldsymbol{S}^{(0)} = \begin{bmatrix} \boldsymbol{x}_{11} & \boldsymbol{x}_{12} & \boldsymbol{x}_{13} & \boldsymbol{x}_{14} & \dots \\ \boldsymbol{x}_{21} & \boldsymbol{x}_{22} & \boldsymbol{x}_{23} & \boldsymbol{x}_{24} & \dots \\ \boldsymbol{x}_{31} & \boldsymbol{x}_{32} & \boldsymbol{x}_{33} & \boldsymbol{x}_{34} & \dots \\ \boldsymbol{x}_{41} & \boldsymbol{x}_{42} & \boldsymbol{x}_{43} & \boldsymbol{x}_{44} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix} \tag{1}$$

represented by a root of a tree T.

For calculation of the first-level spectral components $S^{(0)}$ should be divided into blocks (basic windows) of 2×2 size; according to the former assumptions, the number of such blocks is $2^{2(k-1)}$. Let us take into account one of them, say, the top-left one denoted (according to its upper-left element x_{11}) by (1,1):

$$\boldsymbol{U}_{1,1}^{(1)} = \begin{bmatrix} \boldsymbol{x}_{11} & \boldsymbol{x}_{12} \\ \boldsymbol{x}_{21} & \boldsymbol{x}_{22} \end{bmatrix} \tag{2}$$

The elements of this sub-matrix (as well as of all other similar to it sub-matrices) will be subjected to the following linear transformations:

$$\begin{aligned} y^{(1)}_{11} &= \tfrac{1}{4}(x_{11} + x_{12} + x_{21} + x_{22}), & y^{(1)}_{12} &= -x_{11} + x_{12} - x_{21} + x_{22}\,, \\ y^{(1)}_{21} &= -x_{11} - x_{12} + x_{21} + x_{22}\,, & y^{(1)}_{22} &= -x_{11} + x_{12} + x_{21} - x_{22}\,. \end{aligned} \tag{3}$$

The transformations can be graphically represented by the following 2×2 numerical masks imposed on the elements (pixels) of the block (graphical symbols are used for better visualisation of patterns:

$$\Sigma:\ \begin{matrix} \square & \square \\ \square & \square \end{matrix} \qquad V:\ \begin{matrix} \bullet & \circ \\ \bullet & \circ \end{matrix} \qquad H:\ \begin{matrix} \bullet & \bullet \\ \circ & \circ \end{matrix} \qquad X:\ \begin{matrix} \bullet & \circ \\ \circ & \bullet \end{matrix} \tag{4}$$

here the following weights have been assigned to the symbols: □ = ¼ , ❍ = 1, ● = –1. Covering a sub-matrix by a mask corresponds to assigning the weights to the elements of the sub-matrix and calculating their weighted sum, as follows from the equations (3). Such operation will be called a *scalar product of matrices* $M\circ U^{(1)}_{i,j}$, where M stands for a mask: Σ, V, H or X. The scalar products thus correspond: $\Sigma\circ U$ - to calculation of the average of the elements of a sub-matrix U, $V\circ U$ - to a difference of sums of the elements laying on vertical lines of U, $H\circ U$ - to a difference of sums of elements laying on horizontal lines of U, and $X\circ U$ - to a difference of sums of elements laying on the diagonals of U.

The 1st-level morphological spectrum is given by a block-matrix:

$$S^{(1)} = [S^{(1)}_{p,q}], \quad p, q \in [1,2,...,2^{k-1}], \tag{5}$$

where

$$S^{(1)}_{p,q} = \begin{bmatrix} \Sigma \circ U^{(1)}_{p,q} & V \circ U^{(1)}_{p,q} \\ H \circ U^{(1)}_{p,q} & X \circ U^{(1)}_{p,q} \end{bmatrix}. \tag{6}$$

It should be pointed out that after the above-described operations $S^{(1)}$ still remains a matrix consisting of $2^k \times 2^k$ components.

Example

A simple numerical example of a 2×2 part of an initial bitmap $S^{(0)}$ and of its 1st-level morphological spectrum $S^{(1)}$ is given below:

$$\boldsymbol{S}^{(0)}_{1,1} = \begin{bmatrix} 16 & 22 \\ 20 & 10 \end{bmatrix}, \qquad \boldsymbol{S}^{(1)}_{1,1} = \begin{bmatrix} 17 & -4 \\ -8 & 16 \end{bmatrix}.$$

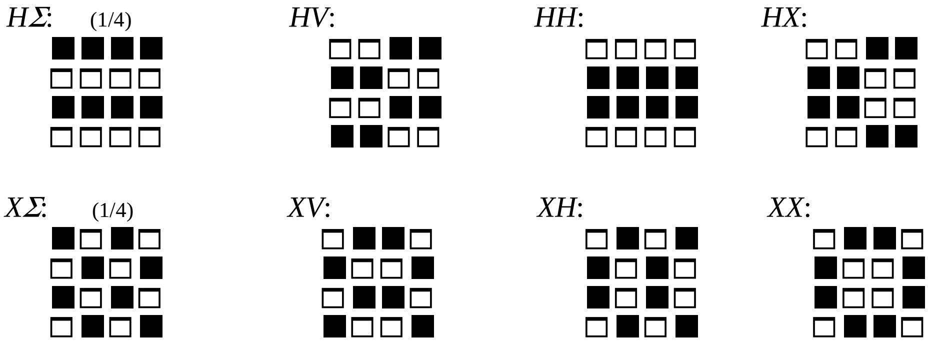
Higher-level morphological spectra are calculated in four times larger basic windows for each next level: 4×4 for 2nd level, 8×8 for 3rd level, etc., up to $2^k\times2^k$ for the k-th level. At each next level the number of basic windows within the testing window is four times lower, while the number of masks becomes four times larger. However, the masks can be generated using combinations of the above described four basic operations, as it will be illustrated by the case of 2nd level spectra. In this case the following 16 masks, denoted by the ordered pairs of symbols Σ, V, H, X, are to be generated:

$$\Sigma\Sigma, \Sigma V, \Sigma H, \Sigma X, V\Sigma, VV, VH, VX, H\Sigma, HV, HH, HX, X\Sigma, XV, XH, XX$$

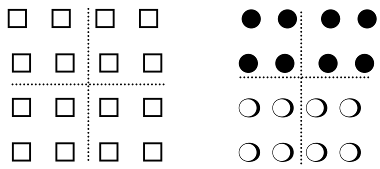
First symbol of a pair denotes a 2×2 mask which should be applied to each of the four squares constituting the 4×4 basic window. Second symbol represents a 2×2 mask which should be extended to a 4×4 mask by doubling each its column and row. Then the a so obtained 4×4 mask should be imposed on the other one and products of the corresponding components of the masks should be calculated. The result is a 4×4 mask represented by the given pair of symbols.

A complete palette of 2×2 masks is shown below:

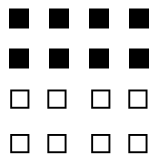
$\Sigma\Sigma$: (1/16)	ΣV: (1/4)	ΣH: (1/4)	ΣX: (1/4)
□□□□	□□■■	■■■■	■■□□
□□□□	□□■■	■■■■	■■□□
□□□□	□□■■	□□□□	□□■■
□□□□	□□■■	□□□□	□□■■
$V\Sigma$: (1/4)	VV:	VH:	VX:
■□■□	□■■□	□■□■	□■■□
■□■□	□■■□	□■□■	□■■□
■□■□	□■■□	■□■□	■□□■
■□■□	□■■□	■□■□	■□□■



Black symbols used above represent sign –, while white symbols represent sign +. The numbers in brackets represent numerical values of weights assigned to the corresponding elements of the mask. For illustration of the above-described method let us take into consideration the mask *ΣH*. The corresponding 2×2 masks *Σ* and *H* are given by eqs. (4). We extend them to the 4×4 size as follows:

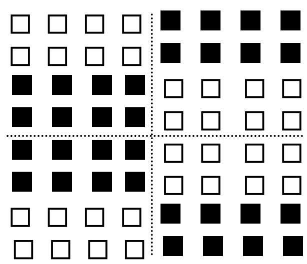


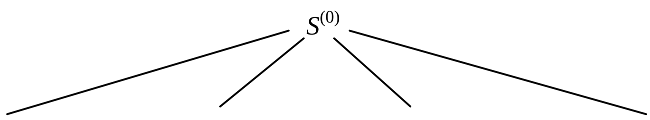
Then, by imposing one mask on the other one, substituting the symbols by their numerical values, calculating products of the corresponding components and representing the numerical values by symbols we obtain the desired mask *ΣH*:



where the weight –¼ is assigned to the black symbols and +¼ to the white ones.

The above-described method can be easily extended on higher-level spectra. For example, the mask *ΣH* being given we can obtain the masks *ΣHΣ, ΣHV, ΣHH* and *ΣHX* , using the method that will be illustrated by the case of *ΣHX*. For this purpose we take into account an 8×8 basic window divided into four 4×4 square blocks. Each block is then covered by the *ΣH* mask. Then we take the 2×2 mask *X* (see eqs. (3)) and we extend it to the desired 8×8 size by four-times repetition of each its column and row. At last, the two 8×8 masks are imposed one on the other one, the products of the corresponding components are calculated and the results are presented in symbolic form. The result is as follows:





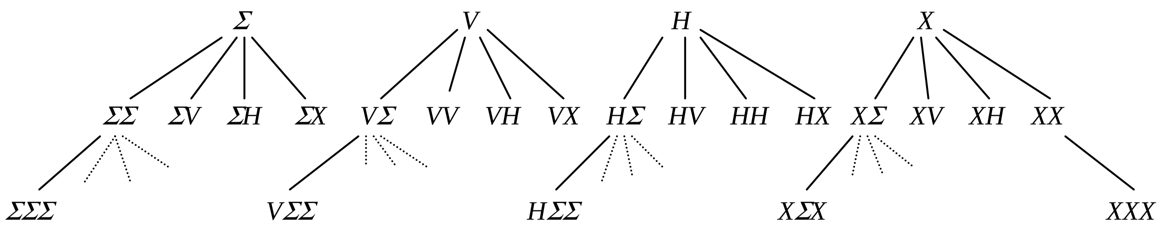


Fig. 1. Tree of morphological spectra.

In similar way the other 63 masks for calculation of $S^{(3)}$ can be obtained.

The structure of morphological spectra can be represented by a rooted tree whose root corresponds to $S^{(0)}$ and the nodes on each level represent the masks used to calculation of spectral components, as it is shown in Fig. 1.

Let us remark that, in general, extension of a spectral-component code by adding a Σ symbol to it causes magnification of its morphological details; adding a *V* or *H* symbol causes a respective elongation of details in vertical or in horizontal direction, adding an *X* symbol generates a chess-desk-form mask.

For satisfactory representation and recognition of a given texture only selected branches and/or sub-branches of the tree should be taken into account. The maximum level of used spectral components should correspond to the size of typical morphological details characterizing the texture under examination. In general, typical feature selection methods should be used to the reduction of the tree of morphological spectral components.

3. CONCLUSIONS

In this report the concept of using morphological spectra to the description of textures has been presented. Morphological spectra allow a multi-scale characterization of textures. They can be easily calculated by using typical masks for any of spectral components. The masks can be generated by simple combinations of four basic operations. All this makes the method useful and handy in practical applications. However, its effectiveness for examination of various types of textures is still to be proven.

REFERENCES

[1]. HADDON J.F., BOYCE J.F. Texture Segmentation and Region Classification by Orthogonal Decomposition of Cooccurence Matrices. Proc. 11th IAPR International Conference on Pattern Recognition, Hague, vol. , pp. 692-695,. IEEE Computer Society Press, Los Alamitos, 1992.

[2]. KULIKOWSKI J.L., WIERZBICKA D. A Method of Microvascular Systems Analysis Based on Statistical Texture Parameter' Evaluation. Biocybernetics and Biomedical Eng., vol. 23. No 3, pp. 21-37, 2003.

[3]. KULIKOWSKI J.L., PRZYTULSKA M., .WIERZBICKA D. Recognition of Textures Based on Analysis of Multilevel Morphological Spectra.(submitted to the IFMBE Conference in Seoul, October, 2006).

[4]. OJALA T., PIETIKAJNENM. Unsupervised Texture Segmentation Using Feature Distributions, Texture Analysis Using Pairwise Interaction Maps, Image Analysis and Processing, 9th International Conference, ICIAP'97, Proc. (Del Bimbo A. ed.), vol. I, pp. 311-318, Florence, 1997.

[5]. SMITHT.G., LANGEG.D. Biological Cellular Morphometry-Fractal Dimensions, Lacunarity and Multifractals. Fractals in Biology and Medicine, vol. II (Losa G.A., Merlini D. et al. Eds.), pp.30-49,Birkhauser, Basel, 1998.

[6]. XIAOHAN Y., YLA-JAASKI J. Unsupervised Texture Segmentation Based on the Modified Markov Random Field Model. Proc. 11th IAPR International Conference on Pattern Recognition, Hague, vol. III. IEEE, pp. 88-91, Computer Society Press, Los Alamitos, 1992.

[7]. ZHU Y.M., GAO Y., GOUTTE R., AMIEL M. Textural Boundary Detection Using Local Spatial Frequency Analysis. Proc. 11th IAPR International Conference on Pattern Recognition, Hague, vol. III, pp. 53-56, IEEE Computer Society Press, Los Alamitos, 1992.