Time-frequency methods, adaptive approximation,
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TIME-FREQUENCY AND TOPOGRAPHICAL ANALYSIS OF SIGNALS

Modern methods of signal analysis are described. Namely, the estimation of time-frequency representation of signals by means of adaptive approximations and directed transfer function defined in the framework of multivariate autoregressive model are delineated and the applications of both methods are given.

1. INTRODUCTION

Technological progress in measurement techniques and increase of the computation power offered new possibilities in respect of application of physiological signals in medicine. In order to exploit in full information contained in these signals appropriate methods of analysis are required. Another challenge in respect of methodology of time series analysis is the rapid progress of the imaging methods such as CAT, PET or especially fMRI. The crucial advantage of signal processing in comparison with above mentioned techniques is the possibility of grasping the dynamic changes in the short time scale. In this respect time-frequency analysis is important. The topographical aspects offered by imaging techniques may be addressed by multichannel signal analysis, which can be also performed in time-frequency. In this paper two methods of signal analysis concerning time-frequency methods and multichannel topographical processing of time series will be described. The applications of the methods will concern mainly brain signals, however they can be used for different kind of signals, not necessarily biomedical signals.

2. ADAPTIVE APPROXIMATIONS BY MATCHING PURSUIT.

Matching Pursuit algorithm (MP) was introduced by Mallat and Zhang [23] and first applied to physiological signal processing by Blinowska and Durka [1]. In order to avoid the effects of dyadic dictionary structure of the original method a new algorithm based on stochastic dictionaries was introduced [5].

The MP method relies on adaptive decomposition of the signal into waveforms from a large and redundant dictionary of functions. A dictionary of basic waveforms can be generated e.g. by scaling, translating and, unlike in wavelet transform, *modulating* window function *g(t)*:

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$$g_I(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{i\xi t} \quad (1)$$

$s>0$ - scale, ξ - frequency modulation, u - translation.
Index $I = (\xi, s, u)$ describes the set of parameters. The dictionaries of windowed Fourier transform and wavelet transform can be derived as subsets of this dictionary, defined by certain restrictions on the choice of parameters. In case of the windowed Fourier transform, the scale s is constant - equal to the window length - and the parameters ξ and u are uniformly sampled. In the case of WT the frequency modulation is limited by the restriction on the frequency parameter $\xi = \xi_0/s$, $\xi_0 = const$.

Finding an optimal approximation of signal by functions from such a large family is a NP-hard problem (computationally intractable). Therefore a suboptimal iterative procedure is applied. In the first step of the iterative procedure we choose the vector g_{I_0} which gives the largest product with the signal $f(t)$. The iterative procedure is repeated; in this way the signal f is decomposed into a sum of time-frequency waveforms, chosen to match optimally the signal's residues R^n [1], [2]:

$$f = \sum_{n=0}^{m} \left\langle R^n f, g_{I_n} \right\rangle g_{I_n} + R^{n+1} f \quad (2)$$

The point at which we should stop the iterations, or equivalently, the number of waveforms in expansion can be chosen individually for each signal based upon mathematical criteria or set arbitrary e.g. as a percentage of energy accounted for.
The highest time-frequency resolution is obtained for functions from Gabor family. The waveforms, or atoms obtained in decomposition procedure are described in terms o their frequency, time occurrence, time span and energy. By adding the Wigner distributions of atoms time-frequency (TF) representation can be easily constructed. Comparison of the TF distributions obtained by different methods revealed that MP provides the highest TF resolution [13].

The method of adaptive approximations provides the parametrisation of all data structures, then one can extract from that large pool of structures the ones belonging to the particular groups defined by clinical criteria e.g. sleep spindles were defined as waveforms of frequency 11 – 15 Hz and time span 0.5 – 2.5 sec, amplitude > 15 μV .

3. APPLICATION OF MP METHOD

In the sleep studies MP approach made possible to distinguish two kinds of spindles, which also differed in topographical features [24]. Temporal evolution of different statistical properties of spindles and SWA was studied; among others the inverse relation in the density of occurrence of SWA and sleep spindles for stage 2 sleep was found. MP procedure allowed for detection of arousals and distinction of deep sleep stages 3 and 4 based directly upon the classical Rechtshafen & Kales criteria [21]. MP describes in one framework rhythmic and transient structures of the signal in terms compatible with visual analysis - in this way clinical knowledge can be quantified and integrated in the automatic system of clinical diagnosis [21].

High resolution of MP allowed for elucidation of the role of different closely spaced rhythms in the voluntary movement experiments [10]. For the first time phenomenon of ERS/ERD (event related desynchronisation/synchronisation) was shown in the whole time-frequency space, not only in the selected bands. Two components of μ rhyhms of different dynamic characteristics were found.

Other applications of MP algorithm concerned event related responses to weak vibrational stimuli [23], and investigation of the evolution of epileptic seizure [9]. In the last experiment MP method allowed for analysis of entire seizures without requiring segmentation or restrictions to stationary epochs. This made possible clear distinction of the periods of different dynamics during seizure development, which correlated with particular kind of clinical picture of illness with implications for therapy.

Almost a decade of MP applications in analysis of EEG suggests that the method can unify most univariate computational approaches to evaluation of this signal, offering at the same time compatibility with its visual analysis.

The application of MP algorithm to otoacustic emissions (OAE) brought a substantial progress in the understanding of its generation mechanisms. In particular the role of resonance modes in the operation of inner ear was elucidated [13], [15] and different origin of the short lasting and long lasting components of emissions was found. The method also proved it usefulness for identification of hearing disturbances [14].

4. MULTIVARIATE AR MODEL AND DIRECTED TRANSFER FUNCTION

For a multichannel process the information derived from the set of signals is quantitatively different from the information obtained from each signal treated separately. The so called cross -measures describing relation between two channels are widely known. However the information which can be derived from whole set of signals is again different from the bivariate measures. The multivariate AR model allows for treatment of signals simultaneously, not pair-wise.

For a *k*-channel signal a vector of *k* EEG values at every time point *t* can be represented as $X(t)=(X_1(t), X_2(t), ..., X_k(t)$. The MVAR model can be expressed as:

$$\mathbf{X}(t) = \sum_{j=1}^{p} \mathbf{A}(j)\mathbf{X}(t-j) + \mathbf{E}(t) \qquad (3)$$

where ***X(t)*** is the data vector in the time *t*, ***E(t)*** is the vector of white noise values, $\mathbf{A}(i)$ are the model coefficients and *p* is the model order. After transforming the model equation to a frequency domain we get [4], [7]:

$$\mathbf{X}(f) = \mathbf{A}^{-1}(f)\mathbf{E}(f) = \mathbf{H}(f)\mathbf{E}(f) \qquad (4)$$

The ***H(f)*** matrix is called a transfer matrix of the system. From the **H*(f)*** power spectra and coherences may be found [4], [7].

In the literature mainly ordinary coherences defined as normalized cross-spectral elements are used. However coherence between two channels may come from the influence of the third channel. In order to distinguish direct from indirect relations partial coherence was introduced. It is nonzero only when the given relation between channels is direct. If a signal in a given channel can be explained by a linear combination of some other signals of the set, the partial coherence between them will

be low [7]. Multiple coherence describes the amount of common component in the given channel and the rest of the signals set.
Parametric analysis of time series provides a natural tool to describe causal relations. When considering Eq.4 it is easy to see that all the relations between data channels are contained in the transfer matrix ***H***. We have introduced [16] Directed Transfer Function (DTF) which describes causal influence of channel j on channel i at frequency f in the form:

$$\gamma_{ij}^2(f) = \frac{\left|H_{ij}(f)\right|^2}{\sum_{m=1}^{k}\left|H_{im}(f)\right|^2} \tag{5}$$

The above equation defines a normalized version of DTF, which takes values from 0 to 1 producing a ratio between the inflow from channel j to channel i to all the inflows to channel i. Sometimes it is easier to abandon the normalization property and use simply values of elements of transfer matrix $H_{ij}(f)$ which are related to causal connection strength [18].

In [18] it was shown that the non-normalized DTF could be interpreted as Granger causality in a multichannel sense. Granger causality [12] was defined for two channels: we say that channel *X(1)* causes *X(2)* in a Granger sense, if the prediction error of *X(2)* is reduced by inclusion of channel *X(1)*, in another words, future of channel *X(2)* may be predicted by means of the past samples not only from channel *X(2)* but also *X(1).*

If more than two channels are mutually dependent it is of outmost importance to include all the channels of the process in the computational model. It was demonstrated by means of simulations and experimental results that calculation of causal relations by means of bivariate measures leads to erroneous results [3], [20], contrary to DTF which provides correct pattern of causal dependencies. The simulation studies revealed also that DTF is extremely robust to noise, even several times higher than the signal.

In order to fit a linear model to a dataset the data segment must be long enough to fulfill a requirement that the number of fitted parameters must not exceed the number of the data points. In practice, we need several times more data points than the model parameters. The number of MVAR parameters is: pk^2, where p is the model order and k is a number of channels, whereas the number of data points is given by kn, where n is a data length in each channel. When we are interested in the dynamical evolution of the signals the data window cannot be long. We may overcome this difficulty when many realizations of the same stochastic process are available. In the procedure of calculating the model coefficients we apply ensemble averaging over realisations, which increases our data m times where m is number of realisations. In consequence we may use short data segments and by application of sliding window technique compute Short-time Directed Transfer Function - SDTF(f,t) as a function of time, not only frequency. The errors of SDTF are estimated by means of a bootstrap method [6], [18].

DTF can be used as well for estimation of causality in the point processes e.g. in the investigation of the spike trains and their connection with local field potentials. This feature was demonstrated by simulations and experimental applications [18].

5. APPLICATION OF DTF AND SDTF

One of the first applications of the DTF method concerned the localisation of epileptic foci [8]. The DTF method allowed for accurate topographical determination of seizure onset and propagation. As the seizure activity spreads regionally, the DTF method can determine whether the initial focus continues to be the source of epileptic activity or whether other more remote areas become secondary generators.

The DTF method was also applied to the estimation of information transfer during locomotion in experimental animals with chronically implanted electrodes [19]. The results obtained by means of DTF together with evidence from partial coherences, have shown that involved structures are connected with a bi-directional links, which are activated depending on the situation – i.e.: difficulty of the motor task and the phase of movement.

DTF method was a basic tool in the topographic analysis of EEG activity during overnight sleep [17]. In the study signals from 21 electrodes (10-20 standard) were simultaneously evaluated by means of the MVAR model. The results indicted that the EEG activity in the awake state (eyes closed) propagates mainly from the posterior areas. During sleep sources of EEG activity shift toward the frontal areas.

SDTF found application in the study of voluntary movements and their imagination. In the series of experiments the finger movements, imagination of hand movements and both movement and imagination were measured [10], [11], [4]. The results revealed similarities in propagation especially in alpha and beta bands. In the gamma band in case of real movement we observed a burst of activity from the motor areas connected with the finger movement followed by the burst from the frontal areas, in case of imagination the alternating flows from the involved structures were observed, especially they came from primary and supplementary motor areas.

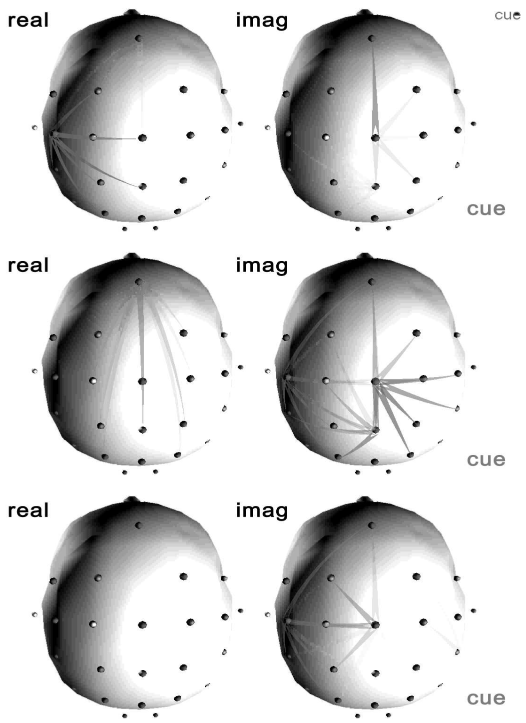

Fig.1. Snapshots of the movie representing propagation of EEG signal during finger movement (left) and imagination of the task (right). From top to the bottom snapshots taken at 0.3s, 1.1s, 1.4s after the cue.

6. CONCLUSIONS.

Both methods described in this paper are complementary — the MP based analysis gives very detailed information about amplitude distribution in a time-frequency domain, while SDTF has lower resolution in time-frequency, but it brings in the information about causal relations between channels. The problem of the determination of directionality and finding causal relationships between time series is at present at the center of interest on many diverse fields e.g. neuroscience, geophysics, economy, sociology. The information about causality is coded in the phases between the channels of a process, however correct causal relationships and directions of signal propagation can only be found when all the interacting channels are evaluated simultaneously. The multichannel data contain rich information which can hardly be accessed by other methods. By the application of appropriate methods of time series analysis this information can be extracted providing us a knowledge about the underlying processes.

BIBLIOGRAPHY

[1] K.J. Blinowska, P.J. Durka. The application of wavelet transform and matching pursuit to the time varying EEG signals. In: Intelligent Engineering Systems through Artificial Neural Networks. Vol.4. pp.535-540. Eds.: C.H.Dagli,B.R.Fernandez. ASME Press, New York, (1994).

[2] K.J. Blinowska, P.J. Durka. Unbiased high resolution method of EEG analysis in time-frequency space. Acta Neurobiologiae Experimentalis. 61:157-174, (2001).

[3] K.J. Blinowska, R. Kus, M. Kaminski, Granger causality and information flow in multivariate processes, Phys Rev E, 70:050902 (2004); Virt J Biol Phys Res, 8(11) (2004).

[4] K.J. Blinowska, M. Kaminski. Multivariate Signal Analysis by Parametric Models. pp. 387-420. In: Handbook of Time Series Analysis, eds. B. Schelter, M. Winterhalder, J. Timmer; Wiley-VCH Verlag. (2006).

[5] Durka P. J., Ircha D., Blinowska K. J. Stochastic time-frequency dictionaries for Matching Pursuit. IEEE Transactions on Signal Processing, 2001; 49:507-510.

[6] B. Efron. Bootstrap methods: another look at the jackknife, Ann Stat, 7, 1-26 (1979).

[7] P.J. Franaszczuk, K.J. Blinowska, M. Kowalczyk, The application of parametric multichannel spectral estimates in the study of electrical brain activity, Biol Cybern, 51, 239-47 (1985).

[8] P.J. Franaszczuk, G.K. Bergey, M. Kaminski, Analysis of mesial temporal seizure onset and propagation using the directed transfer function method, Electroenceph Clin Neurophys, 91, 413-27 (1994).

[9] Franaszczuk P. J., Bergey G. K., Durka P. J., Eisenberg H. M. Time-frequency analysis using the matching pursuit algorithm applied to seizures originating from the mesial temporal lobe. Electroenceph. Clin. Neurophys. 1998; 106:513-521.

[10] J. Ginter Jr., K. J. Blinowska, M. Kaminski, P. J. Durka, Phase and amplitude analysis in time-frequency space — application to voluntary finger movement, J Neurosci Meth, 110, 113-124, (2001).

[11] J. Ginter Jr., K.J. Blinowska, M. Kaminski, P.J. Durka, G. Pfurtscheller, C. Neuper. Propagation of EEG activity in beta and gamma band during movement imagery in human. Methods Inf. Med. 44:106-113, (2005).

[12] C.W.J. Granger, Investigating causal relations in econometric models and cross-spectral methods, Econometrica, 37, 424-38 (1969).

[13] W.W. Jedrzejczak, K.J. Blinowska, W. Konopka, A. Grzanka, P.J. Durka. Identification of otoacoustic emissions components by means of adaptive approximations. Journal of the Acoustic Society of America. 155:2148-2158 (2004).

[14] W.W. Jedrzejczak, K. J. Blinowska, W. Konopka. Time-frequency analysis of transiently evoked otoacoustic emissions of subjects exposed to noise. Hearing Research. 205:249-255, (2005).

[15] W.W. Jedrzejczak, K. J. Blinowska, W. Konopka. Resonant modes in transiently evoked otoacoustic emissions and asymmetries between left and right ear. Journal of the Acoustical Society of America. 119:2226-2231, (2006).

[16] M. Kaminski, K.J. Blinowska, A new method of the description of the information flow in brain structures, Biol Cybern, 65, 203-10 (1991).

[17] M. Kaminski, K.J. Blinowska, W. Szelenberger, Topographic analysis of coherence and propagation of EEG activity during sleep and wakefulness, Electroenceph Clin Neurophys, 102, 216-27 (1997).

[18] M. Kaminski, M. Ding, W. Truccolo, S. Bressler, Evaluating causal relations in neural systems: Granger causality, directed transfer function and statistical assessment of significance, Biol Cybern 2001, 85, 145-57 (2001).

[19] A. Korzeniewska, S. Kasicki, M. Kamiński, K. J. Blinowska, Information flow between hippocampus and related structures during various types of rat's behavior, J Neurosci Meth, 73, 49-60 (1997).

[20] R. Kuś, M. Kamiński, K.J. Blinowska, Determination of EEG activity propagation: pair-wise versus multichannel estimate, IEEE Trans Biomed Eng, 51, 1501-10 (2004).

[21] U. Malinowska, P.J. Durka, K.J. Blinowska, W. Szelenberger, A. Wakarow. Micro- and Macrostructure of Sleep. EEG. IEEE BME Mag. 25:26-31, (2006).

[22] Mallat S. G., Zhang Z. Matching Pursuit with time-frequency dictionaries. IEEE Transactions on Signal Processing 1993; 41:3397-3415.

[23] Żygierewicz J., Kelly E. F., Blinowska K. J., Durka P. J, Folger S. E. Time-Frequency Analysis of Vibrotactile Driving Responses by Matching Pursuit. Journal of Neuroscience Methods 1998; 81:121-129.

[24] Żygierewicz J., Blinowska K. J., Durka P. J., Szelenberger W., Niemcewicz S., Androsiuk W. High resolution study of sleep spindles. Clinical Neurophysiology 1999; 110:2136-2147.